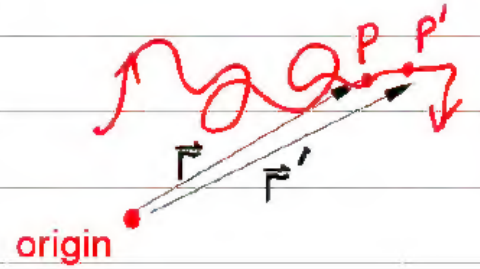
# MIE 100: Lecture #3

## Meriam 2/3: Curvilinear Motion

We consider a particle moving on a curved path in a single plane

E.g. Driving a car in the city




**Define:**

$\vec{r}$ = Vector from origin to point P, at time $t$

$\vec{r}'$ = vector from origin to point P', at time $t'$

where $\vec{r}' = \vec{r} + \Delta\vec{r}$

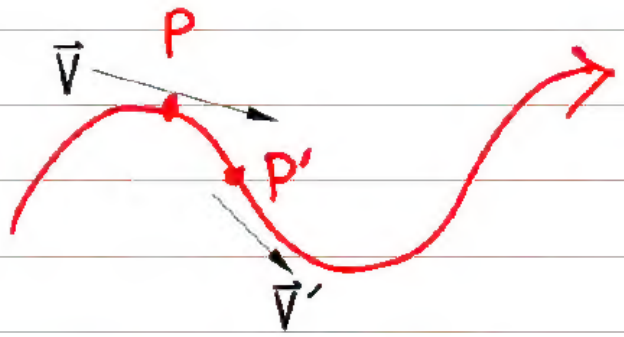
$t' = t + \Delta t$

$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{\vec{r}}$$

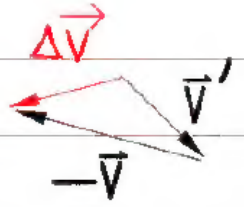
$$v = |\vec{v}| = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \dot{\vec{v}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

Eg.

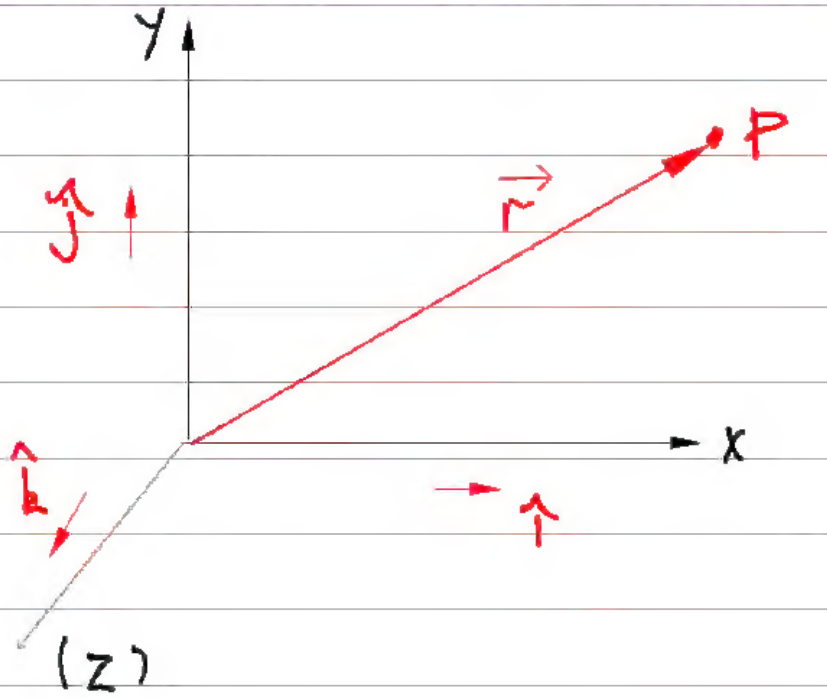



$$\Delta \vec{v} = \vec{v}' - \vec{v}$$




We will use various coordinate systems to transform two-dimensional (2D) vector problems into a system of scalar problems.

# Meriam 2/4: Rectangular Coordinates (x-y-z)




$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left[x\hat{i} + y\hat{j}\right]$$

$$\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j} = v_x\hat{i} + v_y\hat{j}$$

$$\vec{a} = \dot{\vec{v}} = \ddot{\vec{r}} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$$

**Note:**

(i) Right-handed coordinate system

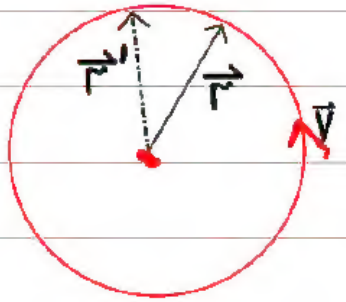
(ii) x, y, z have units (m), +/- signs

(iii) $\hat{i}$, $\hat{j}$, $\hat{k}$ do not vary with time

## Key Point :

The time derivative of a vector is non-zero if its magnitude and/or direction change with time

Eg.




circular motion at 50 m/s.

- $\vec{r}$ is time-varying
- speed $= |\vec{v}| =$ constant
- $\vec{v}$ is time varying (its direction changes)

$$\rightarrow \quad \vec{a} = \frac{d\vec{v}}{dt} \neq 0$$

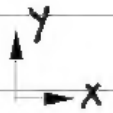
$$\text{speed } v = |\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

where $v_x = \dot{x}$ , $v_y = \dot{y}$ (+/− , units)

$\|_y$, $a_x = \ddot{x}$ , $a_y = \ddot{y}$ , $a_z = \ddot{z}$

The x-y coordinate system is particularly useful when the motion is easily separated into two perpendicular directions

Eg: short-range projectile problems



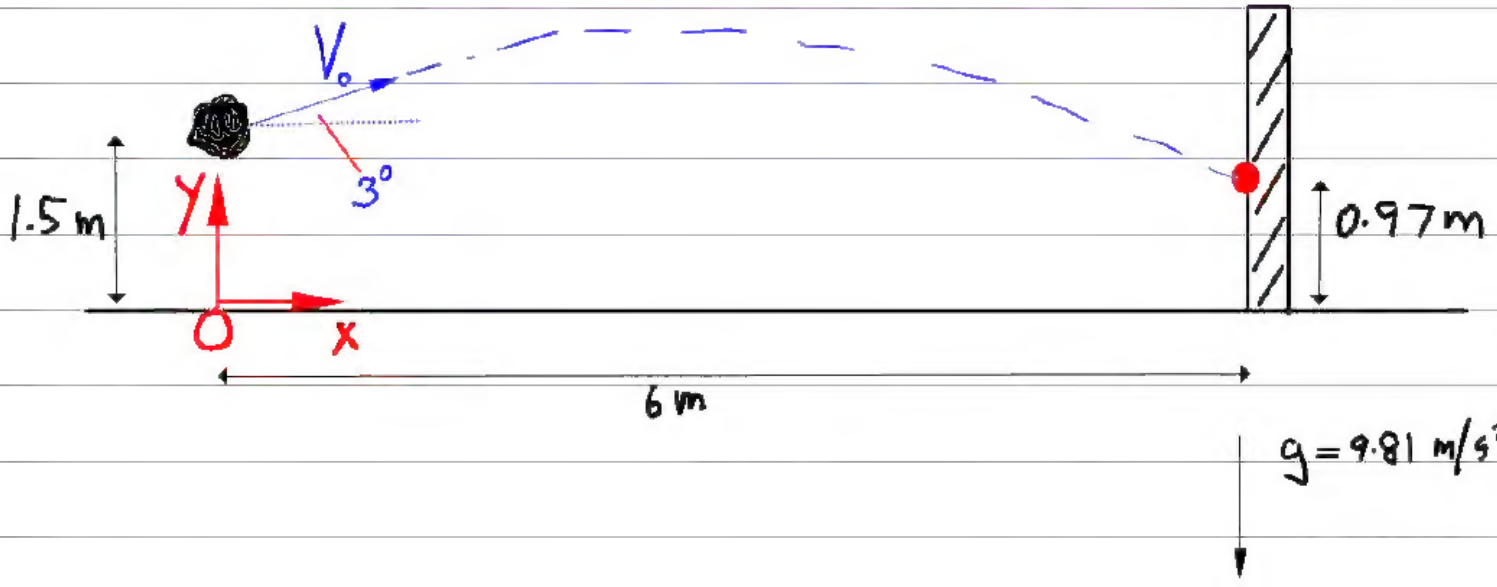

$a_y = -g$
$a_x = 0$

(ignore air resistance)

Always define the coordinate system, & be consistent:

$a_y, V_y, F_y, y$ ↑+
$a_x, V_x, F_x, x$ →+

Eg. Find $V_0$ s/t ball hits red target, if it is thrown at an angle 3° as shown.



## x-direction:

$a_x = 0 \rightarrow V_x(t) = (V_0)_x = V_0 \cos(3°)$

At target •, $x = 6m = x_0 + (V_0)_x t + \frac{1}{2} a_x t^2$

$x_0 = 0$, $(V_0)_x = V_0 \cos(3°)$, $a_x = 0$

Two Unknowns: $V_0$, $t$ $\rightarrow$ $t = 6 / V_0 \cos(3°)$

## y-direction:

At target •, $y = y_0 + (V_0)_y t + \frac{1}{2} a_y t^2$

$y = 0.97$, $y_0 = 1.5$, $(V_0)_y = V_0 \sin(3°)$, $a_y = -9.81$

Sub $t = 6 / V_0 \cos(3°)$

$$\rightarrow 0.97 = 1.5 + \left(V_0 \sin(3°)\right)\left(\frac{6}{V_0 \cos 3°}\right) + \left(\frac{1}{2}\right)(-9.81)\left(\frac{6}{V_0 \cos 3°}\right)^2$$

$\rightarrow V_0 = +/- 14.5$

$\rightarrow$ $\boxed{V_0 = 14.5 \text{ m/s}}$

We broke down vector problem into 2 scalar eqns.

# Meriam 2/5 : Normal-Tangential Coordinates

Consider a different kind of problem:

You are driving a car in random directions and speeds in a huge parking lot

- No reference point to monitor your location
- You don't really know the direction in which you are heading

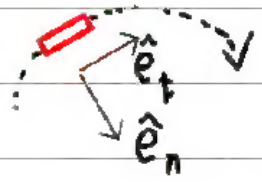
- But relative to the car itself, there are two directions that have physical meaning:

1/ The direction in which you are travelling
ie, the direction of your current velocity

→ the direction that is tangent to your motion

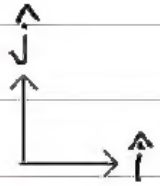
2/ The direction perpendicular or normal to your motion.

- You control acceleration in the tangential direction with the accelerator & brakes
- You control acceleration in the normal direction wit the steering wheel.
- We define unit vectors $\hat{e}_n$ $\hat{e}_t$




- Note that $\hat{e}_n$ and $\hat{e}_t$ depend on the instantaneous velocity, and will change whenever the car changes direction.

This is very different from $\hat{i}$ & $\hat{j}$, which do NOT vary with time



# Summary For lecture 3

- We defined $\vec{r}$, $\vec{v} = \dot{\vec{r}}$, $\vec{a} = \ddot{\vec{r}}$ for curvilinear motion

- Reviewed x-y coordinate system including origin, axes, $\hat{i}$ & $\hat{j}$

- Introduced n-t coordinate system, including unit vectors $\hat{e}_n$, $\hat{e}_t$

## To prepare for next lecture

- Read Meriam 2/6

- Review handout on "Measurement of Angles in Radians"